1 $x_{n+1} = \sqrt[3]{3x_n + 7}$

Use a starting value of $x_1 = 2$ to work out a solution to $x = \sqrt[3]{3x + 7}$

Give your answer to 3 decimal places.

[3 marks]

$x_1 = 2$

$x_2 = \sqrt[3]{3(2) + 7} = \sqrt[3]{13}$ 1

$= 2.351\ldots\ldots$

$x_3 = \sqrt[3]{3(2.351\ldots + 7}$

$= 2.413\ldots\ldots$

$x_4 = \sqrt[3]{3(2.413\cdots + 7}$ 1

$= 2.4238\cdots$

$x_5 = \sqrt[3]{3(2.4238\ldots + 7}$

$= 2.4256\ldots$

$x_6 = \sqrt[3]{3(2.4256\ldots + 7}$

$= 2.4259$

Answer 2.426 1

2 A sphere has radius r cm

An approximate value of r can be found using the iterative formula

$$\boxed{r_{n+1} = \sqrt{\frac{239}{r_n}}}$$

The starting value is $r_1 = 7$

2 (a) Work out the values of r_2 and r_3

[2 marks]

$r_2 = \sqrt{\frac{239}{7}} = 5.843\ldots.$

$r_3 = \sqrt{\frac{239}{5.843\ldots}} = 6.395\ldots.$

$r_2 =$ 5.843… 1

$r_3 =$ 6.395… 1

2 (b) Continue the iteration to work out the radius to 1 decimal place.

[1 mark]

$r_4 = \sqrt{\frac{239}{6.395\ldots}} = 6.113\ldots.$

$r_5 = \sqrt{\frac{239}{6.113\ldots}} = 6.252\ldots$

$r_6 = \sqrt{\frac{239}{6.252\ldots}} = 6.182\ldots$

$r_7 = 6.217$

Answer 6.2 1 cm

3 An approximate value of a root of an equation, x, can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is $x_1 = 4$

3 (a) Work out the values of x_2 and x_3

[2 marks]

$x_2 = \sqrt[3]{5(4)^2 - 2(4) - 3} = 4.10$ (1)

$x_3 = \sqrt[3]{5(4.1)^2 - 2(4.1) - 3} = 4.176 = 4.18$ (2 d.p)

(1)

$x_2 =$ 4.10

$x_3 =$ 4.18

3 (b) By continuing the iteration, show that the value of x is more than 4.25

[1 mark]

$x_4 = 4.23$ $x_7 = 4.33$ $x_{10} = \boxed{4.37}$

$x_5 = 4.28$ $x_8 = 4.34$ $x_{11} = \boxed{4.37}$

$x_6 = 4.31$ $x_9 = 4.36$

(1)

4 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{20}{u_n + 3} \quad \text{where} \quad u_1 = 1$$

Work out u_3

$u_2 = \frac{20}{4} = 5$ $u_3 = \frac{20}{8} = \frac{5}{2}$

Circle your answer.

[1 mark]

$\frac{40}{11}$ $\frac{5}{2}$ (1) 7 5

5

$$x_{n+1} = 5 - \frac{1}{x_n}$$

Use $x_1 = 1$ to work out an approximate solution to $x = 5 - \frac{1}{x}$

Give your answer to 4 significant figures.

[3 marks]

$n = 1, \quad x_2 = 5 - \frac{1}{x_1} = 5 - \frac{1}{1} = 4$

$n = 2, \quad x_3 = 5 - \frac{1}{x_2} = 5 - \frac{1}{4} = 4.75$

$n = 3, \quad x_4 = 5 - \frac{1}{x_3} = 5 - \frac{1}{4.75} = 4.78947\ldots$

3

$n = 4, \quad x_5 = 5 - \frac{1}{x_4} = 5 - \frac{1}{4.78947\ldots} = 4.79121\ldots$

$n = 5, \quad x_6 = 5 - \frac{1}{x_5} = 5 - \frac{1}{4.79121\ldots} = 4.79128\ldots$

$x =$ 4.791